Simplex Method Solution

# Problem:

Maximize:  
  
z = 2x1 - x2 + 2x3  
  
Subject to:  
  
2x1 + x2 ≤ 10  
x1 + 2x2 - 2x3 ≤ 20  
x2 + 2x3 ≤ 5  
  
x1, x2, x3 ≥ 0

# Step 1: Convert inequalities to equalities by adding slack variables:

2x1 + x2 + s1 = 10  
x1 + 2x2 - 2x3 + s2 = 20  
x2 + 2x3 + s3 = 5  
  
where s1, s2, s3 ≥ 0.

# Step 2: Set up the initial simplex tableau:

| Basis | x1 | x2 | x3 | s1 | s2 | s3 | RHS |  
|-------|----|----|----|----|----|----|-----|  
| s1 | 2 | 1 | 0 | 1 | 0 | 0 | 10 |  
| s2 | 1 | 2 | -2 | 0 | 1 | 0 | 20 |  
| s3 | 0 | 1 | 2 | 0 | 0 | 1 | 5 |  
| z | -2 | 1 | -2 | 0 | 0 | 0 | 0 |

# Step 3: Identify entering and leaving variables:

Entering variable: x1 (-2)  
Leaving variable: s1 (minimum ratio 5)

# Step 4: Pivot operation:

Updated tableau after pivoting around (Row 1, Column x1):  
  
  
| Basis | x1 | x2 | x3 | s1 | s2 | s3 | RHS |  
|-------|----|----|----|----|----|----|-----|  
| x1 | 1 | 0.5| 0 | 0.5| 0 | 0 | 5 |  
| s2 | 0 | 1.5| -2 | -0.5| 1 | 0 | 15 |  
| s3 | 0 | 1 | 2 | 0 | 0 | 1 | 5 |  
| z | 0 | 2 | -2 | 1 | 0 | 0 | 10 |

# Step 5: Repeat - Identify entering and leaving variables:

Entering variable: x3 (-2)  
Leaving variable: s3 (minimum ratio 2.5)

# Step 6: Pivot operation:

Updated tableau after pivoting around (Row 3, Column x3):  
  
  
| Basis | x1 | x2 | x3 | s1 | s2 | s3 | RHS |  
|-------|----|----|----|----|----|----|-----|  
| x1 | 1 | 0.5| 0 | 0.5| 0 | 0 | 5 |  
| s2 | 0 | 2.5| 0 | -0.5| 1 | 1 | 20 |  
| x3 | 0 | 0.5| 1 | 0 | 0 | 0.5| 2.5 |  
| z | 0 | 3 | 0 | 1 | 0 | 1 | 15 |

# Step 7: Check Optimality:

All coefficients in z-row are non-negative. Thus, the current solution is optimal.

# Step 8: Final Solution:

x1 = 5  
x3 = 2.5  
x2 = 0  
  
Maximum value of z = 15.  
  
Answer:  
z = 15 at (x1, x2, x3) = (5, 0, 2.5).